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ORIGINAL PAPER

Exact solutions to magnetogasdynamic equations in Lagrangian coordinates

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Abstract The Lie group of point transformations, which leave the equations for a simplified model of one dimensional ideal gas in magnetogasdynamics invariant, are used to obtain some exact solutions for the governing system of hyperbolic partial differential equations (PDEs). Similarity variables which reduces the governing system of PDEs into system of ordinary differential equations (ODEs) are determined through the transformations. The resulting ODEs are solved analytically to obtain some exact solutions that exhibits space-time dependence. Further, we study the propagation of weak discontinuity through a state characterized by one of the solutions.

Keywords Lie group analysis · Exact solution · Weak discontinuity

1 Introduction

In the recent past, analysis of magnetogasdynamics has been the subject of great interest both from mathematical and physical point of view due to it's applications in variety of fields such as astrophysics, nuclear science, engineering physics and studies of magnetogasdynamic effects in problems where chemical reaction plays an important role such as propagation and structure of detonation waves; flows behind detonations; ionization in shocks and detonations etc. As a discipline, magnetogasdynamics bring together a variety of equations of motion for a chemically reacting gas in

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the flow phenomena. A mathematical model can be formulated for such flow of inviscid thermally conducting/ non-conducting, compressible/incompressible, ideal/non-ideal fluid of infinite electrical conductivity in the presence of a magnetic field. Magnetogasdynamics, the science concerned with the mutual interaction between electromagnetic field and flow of electrically conducting gas, offers promising advances in flow control and propulsion of future hypersonic vehicles. Since the full governing system for magnetogasdynamics is highly nonlinear and complicated, it is necessary to study the various simplified models. Today engineering and science researchers routinely confront problems in gaining a better understanding of such mathematical models formulated in terms of nonlinear differential equations. The exact solutions of such system of partial differential equations (PDEs) provide useful information towards our understanding of the complex physical phenomena involved in it. Besides, its own intrinsic interest, these explicit exact solutions may be used for modeling, designing, testing numerical procedure for solving special initial and/or boundary value problems. Indeed for nonlinear systems, involving discontinuities such as shocks, we do not have the luxury of complete exact solutions, and for analytical work we have to rely on some approximate analytical or numerical methods. Lie symmetry analysis which was developed by Sophius Lie [1–3], is one of the extensively used methods to find point transformations which leaves the given PDEs invariant, and allow to determine some particular as well as exact solutions for the governing system of PDEs. Usually, one can determine the corresponding similarity solutions, by solving the over determined system obtained from the original system, from which classes of exact solutions may be recovered. Several researchers has illustrated the advantages of applications of Lie group analysis for investigating nonlinear differential equations [4–6]. A further contributions of this technique may be found in [7,8]. Similarity solutions to the system of PDEs governing three-dimensional Euler equations using Lie group of transformations can be seen in [9]. The work in [10] accounts symmetry reduction, group invariant solutions and some exact solutions of (2+1)-dimensional Jaulent–Miodek equation. Propagation of weak discontinuities in one-dimensional ideal isentropic magnetogasdynamics can be found in [11]. The interaction of a weak discontinuity wave with the elementary waves of the Riemann problem for the one-dimensional Euler equations governing the flow of ideal polytropic gases is investigated in [12]. Reductions of Euler equations for incompressible fluids in two dimensions were obtained in [13], whilst evolution of weak discontinuities in a two-dimensional steady supersonic flow of a non-ideal radiating gas is studied in [14].

In this paper, the equations for a simplified model of one dimensional ideal gas in magnetogasdynamics is considered. Lie group analysis is performed and certain classes of exact solutions to the governing system of PDEs are obtained. With the exact solution in hand, the behavior of evolution of weak discontinuity is studied.

2 Group analysis

The one dimensional adiabatic flow of an ideal, inviscid, perfectly conducting compressible fluid subject to transverse magnetic field (in Lagrangian coordinates) can be written as follows [15]:

$$\begin{aligned} &\tau_t - u_x = 0, \\ &u_t + p_x - \frac{k^2}{\mu\tau^3}\tau_x = 0, \\ &p_t + \frac{\gamma p}{\tau}u_x = 0, \end{aligned} \tag{1}$$

where τ, u and p are the specific volume, the velocity and the pressure. μ is the magnetic permeability, k is a positive constant where γ is the adiabatic gas constant with $1 < \gamma < 3$ for most of the gases. The independent variables t and x denote time and space respectively. We investigate the most general Lie group of transformations which leaves the governing system of equations (1) invariant. Considering the Lie group of transformations with independent variables t and x: and dependent variables τ, u and p for the problem and following the straightforward analysis mentioned in [3,5], we obtain the set of infinitesimal transformations as

$$\begin{aligned} &\phi_1 = \alpha_1 + \alpha_2 t, \qquad \phi_2 = \alpha_3 + \alpha_4 x, \qquad \psi_1 = \frac{2(\alpha_2 - \alpha_4)\tau}{3}, \\ &\psi_2 = \alpha_5 + \frac{(\alpha_2 - \alpha_4)u}{3}, \qquad \psi_3 = \frac{4(\alpha_2 - \alpha_4)p}{3}. \end{aligned} \tag{2}$$

where $\alpha_1,\ \alpha_2,\ \alpha_3,\ \alpha_4$ and α_5 are arbitrary constants. The similarity variables, which allows us to reduce the given system of PDEs to the system of ordinary differential equations (ODEs), can be obtained from the characteristic equations considering different cases as given below:

$$\frac{dt}{\phi_1} = \frac{dx}{\phi_2} = \frac{d\rho}{\psi_1} = \frac{du}{\psi_2} = \frac{dp}{\psi_3}. \tag{3}$$

i.e.,

$$\frac{dt}{\alpha_1 + \alpha_2 t} = \frac{dx}{\alpha_3 + \alpha_4 x} = \frac{d\tau}{\frac{2(\alpha_2 - \alpha_4)\tau}{3}} = \frac{du}{\alpha_5 + \frac{(\alpha_2 - \alpha_4)u}{3}} = \frac{dp}{\frac{4(\alpha_2 - \alpha_4)p}{3}}.$$

Case A: $\alpha_3 \neq 0, \alpha_4 \neq 0$ and $\alpha_4 = \alpha_2$.
This case yields the similarity and dependent variables as follows:

$$\xi = (\alpha_3 + \alpha_2 x)^{-1}(\alpha_1 + \alpha_2 t), \quad \tau = R, \quad u = \ln\left((\alpha_1 + \alpha_2 t)^{\frac{\alpha_5}{\alpha_2}} U\right), \quad p = P. \tag{4}$$

Substitution of the variables from (4) in (1) we obtain the reduced system of ODEs as follows

$$\alpha_2 \frac{dR}{d\xi} + \frac{\alpha_2}{U}\xi\frac{dU}{d\xi} - \alpha_5 = 0,$$

$$\frac{\alpha_2}{U}\frac{dU}{d\xi} - \alpha_2\xi\frac{dP}{d\xi} + \frac{k^2\alpha_2}{\mu R^3}\xi\frac{dR}{d\xi} = 0,$$
$$\alpha_2\frac{dP}{d\xi} + \frac{\gamma P}{R}\left(\alpha_5 - \frac{\alpha_2}{U}\xi\frac{dU}{d\xi}\right) = 0. \tag{5}$$

The system of ODEs (5) are solved for $\gamma = 2$ and obtained as

$$R = \frac{\alpha_5}{\alpha_2}\xi, \qquad U = C_1, \qquad P = -\frac{k^2\alpha_2^2}{2\mu\alpha_5^2}\frac{1}{\xi^2}, \tag{6}$$

where C_1 is an integration constant. Further (4) and (6) together produces the solution for (1) as below

$$R = \frac{\alpha_5(\alpha_1 + \alpha_2 t)}{\alpha_2(\alpha_3 + \alpha_2 x)}, \qquad U = \ln\left((\alpha_1 + \alpha_2 t)^{\frac{\alpha_5}{\alpha_2}} C_1\right), \qquad P = -\frac{k^2\alpha_2^2(\alpha_3 + \alpha_2 x)^2}{2\mu\alpha_5^2(\alpha_1 + \alpha_2 t)^2}.$$

Case B: $\alpha_3 \neq 0$ and $\alpha_4 = 0$.
The similarity variable and the dependent variables associated to this case are

$$\xi = (\alpha_1 + \alpha_2 t)^{-1}, \qquad \tau = R\ \exp\left(\frac{2\alpha_2 x}{3\alpha_3}\right), \qquad u = -U\ \exp\left(\frac{-\alpha_2 x}{3\alpha_3}\right) + \frac{3\alpha_5}{\alpha_2},$$
$$p = P\ \exp\left(\frac{-4\alpha_2 x}{3\alpha_3}\right), \tag{7}$$

Usage of (7) in (1) yields the reduced system of ODEs as follows

$$-\alpha_2\xi^2\frac{dR}{d\xi} + \frac{\alpha_2\xi}{\alpha_3}\frac{dU}{d\xi} - \frac{\alpha_2}{3\alpha_3}U = 0,$$
$$\alpha_2\xi^2\frac{dU}{d\xi} + \frac{\alpha_2\xi}{\alpha_3}\frac{dP}{d\xi} - \frac{4\alpha_2}{3\alpha_3}P - \frac{k^2}{\mu R^3}\left(\frac{2\alpha_2}{3\alpha_3}R + \frac{\alpha_2\xi}{\alpha_3}\frac{dR}{d\xi}\right) = 0, \tag{8}$$
$$\alpha_2\xi^2\frac{dP}{d\xi} - \frac{\gamma P}{R}\left(\frac{\alpha_2\xi}{\alpha_3}\frac{dU}{d\xi} - \frac{\alpha_2}{3\alpha_3}U\right) = 0.$$

Assuming $U = 3\alpha_3$ and $\gamma = 1$, we obtain the solution of (8) as

$$R = \frac{1}{\xi}, \qquad U = 3\alpha_3, \qquad P = \frac{-k^2}{2\mu}\xi^2,$$

which in turn gives the solution of (1) as below

$$\tau = (\alpha_1 + \alpha_2 t)\exp\left(\frac{-\alpha_2 x}{3\alpha_3}\right), \qquad u = -3\alpha_3\exp\left(\frac{-\alpha_2 x}{3\alpha_3}\right) + \frac{3\alpha_5}{\alpha_2},$$
$$p = -\frac{k^2}{2\mu}(\alpha_1 + \alpha_2 t)^{-2}\exp\left(\frac{2\alpha_2 x}{3\alpha_3}\right).$$

Case C: $\alpha_3 = 0$ and $\alpha_4 = 0$.
For this case the new similarity variable and the dependent variables are

$$\xi = x, \qquad \tau = R\,(\alpha_1 + \alpha_2 t)^{\frac{2}{3}}, \qquad u = -U\,(\alpha_1 + \alpha_2 t)^{-\frac{1}{3}} + \frac{3\alpha_5}{\alpha_2},$$
$$p = P\,(\alpha_1 + \alpha_2 t)^{-\frac{4}{3}}, \tag{9}$$

The variables in (9) are used in (1), obtained the reduced system of ODEs as follows

$$\begin{aligned} &-\frac{dU}{d\xi} + \frac{2\alpha_2}{3}R = 0,\\ &\frac{dP}{d\xi} - \frac{k^2}{\mu R^3}\frac{dR}{d\xi} + \frac{\alpha_2}{3}U = 0,\\ &\frac{\gamma P}{R}\frac{dU}{d\xi} - \frac{4\alpha_2}{3}P = 0. \end{aligned} \tag{10}$$

We can obtain the solution of (10) by assuming $R = \frac{3}{2\alpha_2}$ and $\gamma = 2$ as

$$R = \frac{3}{2\alpha_2}, \qquad U = \xi, \qquad P = \frac{-\alpha_2}{6}\xi^2,$$

which in turn gives the solution of (1) as below

$$\tau = \frac{3}{2\alpha_2}(\alpha_1 + \alpha_2 t)^{\frac{2}{3}}, \qquad u = x(\alpha_1 + \alpha_2 t)^{-\frac{1}{3}} + \frac{3\alpha_5}{\alpha_2},$$
$$p = \frac{\alpha_2}{6}x^2(\alpha_1 + \alpha_2 t)^{-\frac{4}{3}}. \tag{11}$$

Case D: $\alpha_1 \neq 0$ and $\alpha_2 \neq 0$.
The similarity variable and new dependent variables are

$$\xi = (\alpha_3 + \alpha_4 x)(\alpha_1 + \alpha_2 t)^{\frac{\alpha_4}{\alpha_1}}, \qquad \tau = R\,(\alpha_1 + \alpha_2 t)^{\frac{2(\alpha_2 - \alpha_4)}{3\alpha_2}},$$
$$u = U\,(\alpha_1 + \alpha_2 t)^{\frac{(\alpha_4 - \alpha_2)}{3\alpha_2}} - \frac{3\alpha_5}{\alpha_4 - \alpha_2}, \qquad p = P\,(\alpha_1 + \alpha_2 t)^{\frac{4(\alpha_4 - \alpha_2)}{3\alpha_2}}. \tag{12}$$

Using (12) in (1), we obtain the following reduced system of ODEs

$$\begin{aligned} &\alpha_4\xi\frac{dR}{d\xi} - \alpha_4\frac{dU}{d\xi} + \frac{2(\alpha_2 - \alpha_4)}{3}R = 0,\\ &\alpha_4\xi\frac{dU}{d\xi} + \alpha_4\frac{dP}{d\xi} - \frac{\alpha_4 k^2}{R^3}\frac{dR}{d\xi} + \frac{(\alpha_2 - \alpha_4)}{3}U = 0,\\ &\alpha_4\xi\frac{dP}{d\xi} + \frac{\alpha_4\gamma P}{R}\frac{dU}{d\xi} + \frac{4(\alpha_2 - \alpha_4)}{3}P = 0, \end{aligned}$$

which on solving for $\gamma = 2$, $R = \alpha_4$ and $\alpha_2 = -2\alpha_4$, we obtain the solution as

$$U = -2\alpha_4\xi, \qquad P = C_2, \tag{13}$$

where C_2 is an arbitrary integration constant. Combining (13) and (12) produces solution of (1) as

$$\tau = \alpha_4(\alpha_1 + \alpha_2 t), \qquad u = -2\alpha_4(\alpha_3 + \alpha_4 x) + \frac{\alpha_5}{\alpha_4}, \qquad p = C_2(\alpha_1 + \alpha_2 t)^{-2}.$$

Case E: $\alpha_1 = 0$ and $\alpha_2 = 0$.
For this case we obtained the similarity variable and the dependent variables as

$$\xi = t, \qquad \tau = R\ (\alpha_3 + \alpha_4 x)^{-\frac{2}{3}}, \qquad u = U\ (\alpha_3 + \alpha_4 x)^{\frac{1}{3}} - \frac{3\alpha_5}{\alpha_4},$$
$$p = P\ (\alpha_3 + \alpha_4 x)^{\frac{4}{3}},$$

which reduces (1) to system of ODEs as

$$\begin{aligned} &\frac{dR}{d\xi} + \frac{\alpha_4}{3}U = 0, \\ &\frac{dU}{d\xi} + \frac{4\alpha_4}{3}P + \frac{2\alpha_4 k^2}{3\mu R^2} = 0, \\ &\frac{dP}{d\xi} + \frac{\gamma\alpha_4 P}{3R}U = 0. \end{aligned} \tag{14}$$

Further, for $U = A =$ constant and $\gamma = 2$ we obtain the solution of (14) as

$$R = \frac{A\alpha_4\xi}{3} + C_3, \qquad U = A \qquad P = \frac{-9k^2}{2\mu}(3C_3 + A\alpha_4\xi)^{-2},$$

where C_3 is an arbitrary integration constant. The corresponding solution of (1) is give as

$$\tau = \frac{1}{3}(A\alpha_4 t + 3C_3)(\alpha_3 + \alpha_4 x)^{-\frac{2}{3}}, \qquad u = A(\alpha_3 + \alpha_4 x)^{\frac{1}{3}} - \frac{3\alpha_5}{\alpha_4},$$
$$p = \frac{-9k^2}{2\mu}(A\alpha_4 t + 3C_3)^{-2}(\alpha_3 + \alpha_4 x)^{\frac{4}{3}}.$$

Case F: $\alpha_2 = 0$ and $\alpha_4 = 0$.
The similarity variable and the dependent variables are

$$\xi = x - \frac{\alpha_3}{\alpha_1}t, \qquad \tau = R, \qquad u = \frac{\alpha_5}{\alpha_1}t + U, \qquad p = P. \tag{15}$$

Usage of (15) in (1) yields the reduced system of ODEs as follows

$$\begin{aligned}
&-\frac{\alpha_3}{\alpha_1}\frac{dR}{d\xi}-\frac{dU}{d\xi}=0,\\
&-\frac{\alpha_3}{\alpha_1}\frac{dU}{d\xi}+\frac{dP}{d\xi}-\frac{k^2}{\mu R^3}\frac{dR}{d\xi}+\frac{\alpha_5}{\alpha_1}=0,\\
&-\frac{\alpha_3}{\alpha_1}\frac{dP}{d\xi}+\frac{\gamma P}{R}\frac{dU}{d\xi}=0.
\end{aligned}\tag{16}$$

Assuming $\alpha_3=\alpha_5$ and $\gamma=2$, we obtain the solution of (16) as

$$R=-\frac{\alpha_1}{\alpha_3}\xi,\qquad U=\xi,\qquad P=\frac{-\alpha_3^2k^2}{2\mu\alpha_1^2}\xi^2,$$

which in turn gives the solution of (1) as below

$$\tau=\left(t-\frac{\alpha_1}{\alpha_3}x\right),\qquad u=x,\qquad p=-\frac{k^2}{2\mu}\left(\frac{\alpha_1}{\alpha_3}x-t\right)^{-2}.$$

3 Evolution of weak discontinuity

The governing hyperbolic system (1) can be written in the matrix form as:

$$V_t+MV_x=0,\tag{17}$$

where $V=(\tau,u,p)^T$ is column vector with superscript T denoting transposition, while M is a matrix with elements $M_{11}=M_{22}=M_{33}=M_{13}=M_{31}=0$, $M_{12}=-1$, $M_{21}=-\frac{k^2}{\mu\tau^3}$, $M_{23}=1$, $M_{32}=\frac{\gamma p}{\tau}$. The matrix M has the eigenvalues

$$\lambda_1=-w,\qquad \lambda_2=0,\qquad \lambda_3=w$$

where $w=\sqrt{\frac{k^2}{\mu\tau^3}+\frac{\gamma p}{\tau}}$ with the corresponding left and right eigenvectors

$$\begin{aligned}
l_1&=\left(\frac{k^2}{\mu\tau^3},w,-1\right),\qquad r_1=\left(-1,-w,\frac{\gamma p}{\tau}\right)^T,\\
l_2&=\left(\frac{\gamma p}{\tau},0,1\right),\qquad r_2=\left(1,0,\frac{k^2}{\mu\tau^3}\right)^T,\\
l_3&=\left(-\frac{k^2}{\mu\tau^3},w,1\right),\qquad r_3=\left(-1,w,\frac{\gamma p}{\tau}\right)^T.
\end{aligned}\tag{18}$$

Let us consider that the weak discontinuity is propagating along the characteristic curve determined by $\frac{dx}{dt}=\lambda_3$ originating from the point (x_0,t_0). Then the transport

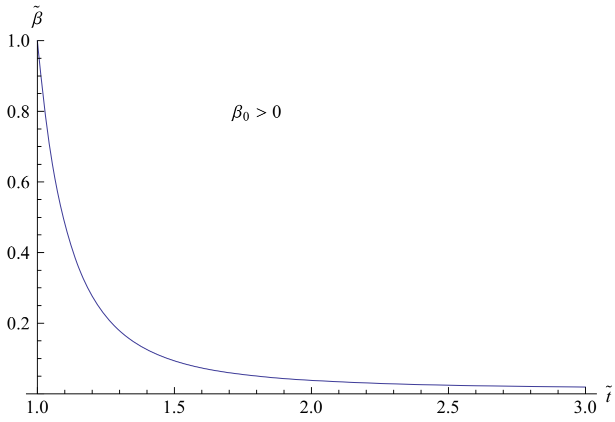

Fig. 1 Behavior of $\tilde{\beta}$ with $\tilde{t}$ for $\beta_0 > 0$

equation for the weak discontinuity across the third characteristic of a hyperbolic system of equations is given by [16]:

$$l_3\left(\frac{d\Lambda}{dt} + (V_x + \Lambda)(\nabla\lambda_3)\Lambda\right) + ((\nabla l_3)\Lambda)^T\frac{dV}{dt} + (l_3\Lambda)((\nabla\lambda_3)V_x + (\lambda_3)_x) = 0, \tag{19}$$

where Λ, denotes the jump in V_x across the weak discontinuity, is collinear to right eigenvector r_3, *i.e* $\Lambda = \beta(t)r_3$ with $\beta(t)$ is the amplitude of the weak discontinuity and $\nabla = \left(\frac{\partial}{\partial\tau}, \frac{\partial}{\partial u}, \frac{\partial}{\partial p}\right)$. Substitution of (11) and (18) along with Λ, λ_3 in (19) gives the following Bernoulli type of equation for the amplitude $\beta(t)$

$$\frac{d\beta}{dt} + \Psi_1(x,t)\beta^2 + \Psi_2(x,t)\beta = 0, \qquad \frac{dx}{dt} = w, \tag{20}$$

where

$$\Psi_1(x,t) = \frac{-\sqrt{2}\alpha_2\left(8\alpha_2k^2 - \gamma(\gamma+1)\mu x^2\right)}{\sqrt{3}\sqrt{\mu}(\alpha_1+\alpha_2 t)^{\frac{5}{3}}\left(8\alpha_2k^2 - 3\gamma\mu x^2\right)},$$

$$\Psi_2(x,t) = \frac{2\alpha_2\sqrt{\mu}x}{\sqrt{3}(\alpha_1+\alpha_2 t)\sqrt{8\alpha_2k^2 - 3\gamma\mu x^2}} + \frac{\left(16\alpha_2k^2 - \gamma(2\gamma+3)\mu x^2\right)}{2(\alpha_1+\alpha_2 t)^{\frac{1}{3}}\left(8\alpha_2k^2 - 3\gamma\mu x^2\right)}.$$

The solution of (20) can be written in quadrature form as $\beta(t) = \frac{\beta_0 S(t)}{1+\beta_0 Q(t)}$ where $S(t) = exp\left(\int_{t_0}^{t} -\Psi_2(x(s),s)ds\right)$ and $Q(t) = \int_{t_0}^{t}\Psi_1(x(t^{'}),t^{'})exp\left(\int_{t_0}^{t^{'}} -\Psi_2(x(s),s)ds\right)dt^{'}$. For the functions Ψ_1 and Ψ_2, given as above, we find that both the integrals $S(t)$ and $Q(t)$ are finite and continuous on $[t_0,\infty)$. From Fig. 1 it is obvious that, for $\beta_0 > 0$, which corresponds to the expansion wave, as $t \to \infty$ the

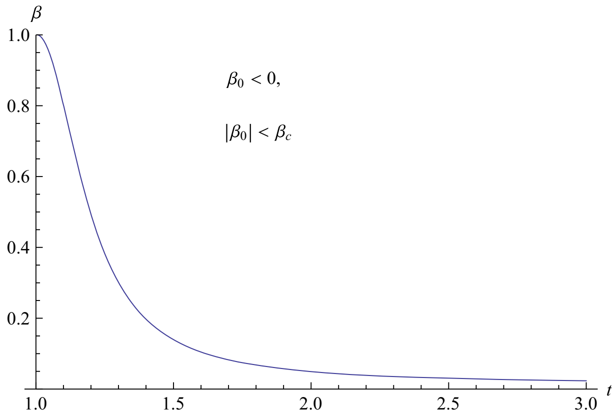

Fig. 2 Behavior of $\tilde{\beta}$ with $\tilde{t}$ for $\beta_0 < 0$ and $|\beta_0| < \beta_c$

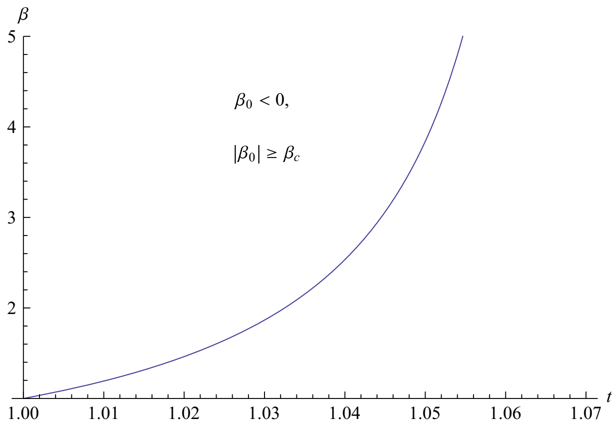

Fig. 3 Behavior of $\tilde{\beta}$ with $\tilde{t}$ for $\beta_0 < 0$ and $|\beta_0| \geq \beta_c$

wave decays and dies out eventually. If $\beta_0 < 0$, then there exist a positive quantity $\beta_c > 0$ such that $|\beta_0| < \beta_c$, the wave decays and dies out and the situation is illustrated in Fig. 2. However, for $\beta_0 < 0$ and $|\beta_0| \geq \beta_c$, there exist a finite time t_c given by the solution of $Q(t_c) = \frac{1}{|\beta_0|}$ such that $|\beta_c| \to \infty$ as $t \to t_c$; this means that when the amplitude of the incident discontinuity exceeds the critical value in magnitude, the wave culminates into a shock in a finite time which can be observed in Fig. 3.

4 Conclusions

The Lie symmetry groups are used to transform the governing system of PDEs to a system of ODEs. Further, the reduced system of ODEs are solved and some exact solutions are derived. These exact solutions to mathematical equations play an important role in the proper understanding of qualitative features of many phenomena and processes in various areas of natural science. Exact solutions of nonlinear differential equations graphically demonstrate and allow unraveling the mechanisms of many complex non-linear phenomena such as spatial localization of transfer processes, multiplicity or absence of steady states under various conditions, existence of peaking regimes, and many others. Furthermore, simple solutions are often used in teaching many courses as specific examples illustrating basic tenets of a theory that admit mathematical formulation such as heat and mass transfer theory, hydrodynamics, gas dynamics, wave theory and other fields. Even those special exact solutions that do not have a clear physical meaning can be used as test problems to verify the consistency and estimate errors of various numerical, asymptotic, and approximate analytical methods. Furthermore, the behavior of weak discontinuity has been discussed across the solution curve which is well illustrated by the Fig. 1, 2, 3. For $\beta_0 > 0$ or $\beta_0 < 0$ and $|\beta_0| < \beta_c$, in both the cases the wave decays and dies out eventually, which has been well observed in Figs. 1 and 2. For $\beta_0 < 0$ and $|\beta_0| \geq \beta_c$ shows the appearance of shock and the situation is observed in Fig. 3.

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